ALGEBRA
1.0 Students identify and use the arithmetic properties of subsets of integers and rational, irrational, and real numbers, for the four basic arithmetic operations where applicable*:
1.1 Students use properties of numbers to demonstrate whether assertions are true or false.
2.0 Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents.*
3.0 Students solve equations and inequalities involving absolute values.
4.0 Students simplify expressions before solving linear equations and inequalities in one variable, such as $3(2x-5) + 4(x-2) = 12$.
5.0 Students solve multistep problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.
6.0 Students graph a linear equation and compute the x- and y-intercepts (e.g., graph $2x + 6y = 4$). They are also able to sketch the region defined by linear inequality (e.g., they sketch the region defined by $2x + 6y < 4$).
7.0 Students verify that a point lies on a line, given an equation of the line. Students are able to derive linear equations by using the point-slope formula.
8.0 Students understand the concepts of parallel lines and perpendicular lines and how their slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.
9.0 Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.
10.0 Students add, subtract, multiply, and divide monomials and polynomials. Students solve multistep problems, including word problems, by using these techniques.
11.0 Students apply basic factoring techniques to second- and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.
12.0 Students simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms.
13.0 Students add, subtract, multiply, and divide rational expressions and functions. Students solve both computationally and conceptually challenging problems by using these techniques.
14.0 Students solve a quadratic equation by factoring or completing the square with B equal to a whole even number and A equal to 1.
15.0 Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.
16.0 Students understand the concepts of a relation and a function, determine whether a given relation defines a function, and give pertinent information about given relations and functions.
17.0 Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.
18 .0 Students determine whether a relation defined by a graph, a set of ordered pairs, or a symbolic expression is a function and justify the conclusion.

Note: **Bold** denotes an essential standard.

19.0 Students know the quadratic formula and are familiar with its proof by completing the square.

20.0 Students use the quadratic formula to find the roots of a second-degree polynomial and to solve quadratic equations.

21.0 Students graph quadratic functions and know that their roots are the x-intercepts.

22.0 Students use the quadratic formula or factoring techniques or both to determine whether the graph of a quadratic function will intersect the x-axis in zero, one, or two points.

23.0 Students apply quadratic equations to physical problems, such as the motion of an object under the force of gravity.

24.0 Students use and know simple aspects of a logical argument:

24.1 Students explain the difference between inductive and deductive reasoning and identify and provide examples of each.

24.2 Students identify the hypothesis and conclusion in logical deduction.

24.3 Students use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion.

25.0 Students use properties of the number system to judge the validity of results, to justify each step of a procedure, and to prove or disprove statements:

25.1 Students use properties of numbers to construct simple, valid arguments (direct and indirect) for, or formulate counterexamples to, claimed assertions.

25.2 Students judge the validity of an argument according to whether the properties of the real number system and the order of operations have been applied correctly at each step.

25.3 Given a specific algebraic statement involving linear, quadratic, or absolute value expressions or equations or inequalities, students determine whether the statement is true sometimes, always, or never.

GEOMETRY

1.0 Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and inductive and deductive reasoning.

2.0 Students write geometric proofs, including proofs by contradiction.

3.0 Students construct and judge the validity of a logical argument and give counterexamples to disprove a statement.

4.0 Students prove basic theorems involving congruence and similarity.

5.0 Students prove that triangles are congruent or similar, and they are able to use the concept of corresponding parts of congruent triangles.

6.0 Students know and are able to use the triangle inequality theorem.

7.0 Students prove and use theorems involving the properties of parallel lines cut by a transversal, the properties of quadrilaterals, and the properties of circles.

8.0 Students know, derive, and solve problems involving the perimeter, circumference, area, volume, lateral area, and surface area of common geometric figures.

9.0 Students compute the volumes and surface areas of prisms, pyramids, cylinders, cones, and spheres; and students commit to memory the formulas for prisms, pyramids, and cylinders.

10.0 Students compute areas of polygons, including rectangles, scalene triangles, equilateral triangles, rhombi, parallelograms, and trapezoids.

Note: **Bold** denotes an essential standard.

11.0 Students determine how changes in dimensions affect the perimeter, area, and volume of common geometric figures and solids.

12.0 Students find and use measures of sides and of interior and exterior angles of triangles and polygons to classify figures and solve problems.

13.0 Students prove relationships between angles in polygons by using properties of complementary, supplementary, vertical, and exterior angles.

14.0 Students prove the Pythagorean theorem.

15.0 Students use the Pythagorean theorem to determine distance and find missing lengths of sides of right triangles.

16.0 Students perform basic constructions with a straightedge and compass, such as angle bisectors, perpendicular bisectors, and the line parallel to a given line through a point off the line.

17.0 Students prove theorems by using coordinate geometry, including the midpoint of a line segment, the distance formula, and various forms of equations of lines and circles.

18.0 Students know the definitions of the basic trigonometric functions defined by the angles of a right triangle. They also know and are able to use elementary relationships between them. For example, $\tan(x) = \sin(x)/\cos(x)$, $(\sin(x))^2 + (\cos(x))^2 = 1$.

19.0 Students use trigonometric functions to solve for an unknown length of a side of a right triangle, given an angle and a length of a side.

20.0 Students know and are able to use angle and side relationships in problems with special right triangles, such as 30°, 60°, and 90° triangles and 45°, 45°, and 90° triangles.

21.0 Students prove and solve problems regarding relationships among chords, secants, tangents, inscribed angles, and inscribed and circumscribed polygons of circles.

22.0 Students know the effect of rigid motions on figures in the coordinate plane and space, including rotations, translations, and reflections.

ALGEBRA II

1.0 Students solve equations and inequalities involving absolute value.

2.0 Students solve systems of linear equations and inequalities (in two or three variables) by substitution, with graphs, or with matrices.

3.0 Students are adept at operations on polynomials, including long division.

4.0 Students factor polynomials representing the difference of squares, perfect square trinomials, and the sum and difference of two cubes.

5.0 Students demonstrate knowledge of how real and complex numbers are related both arithmetically and graphically. In particular, they can plot complex numbers as points in the plane.

6.0 Students add, subtract, multiply, and divide complex numbers.

7.0 Students add, subtract, multiply, divide, reduce, and evaluate rational expressions with monomial and polynomial denominators and simplify complicated rational expressions, including those with negative exponents in the denominator.

8.0 Students solve and graph quadratic equations by factoring, completing the square, or using the quadratic formula. Students apply these techniques in solving word problems. They also solve quadratic equations in the complex number system.

Note: **Bold** denotes an essential standard.

9.0 Students demonstrate and explain the effect that changing a coefficient has on the graph of quadratic functions; that is, students can determine how the graph of a parabola changes as a, b, and c vary in the equation $y = a(x-b)^2 + c$.

10.0 Students graph quadratic functions and determine the maxima, minima, and zeros of the function.

11.0 Students prove simple laws of logarithms.

11.1 Students understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

11.2 Students judge the validity of an argument according to whether the properties of real numbers, exponents, and logarithms have been applied correctly at each step.

12.0 Students know the laws of fractional exponents, understand exponential functions, and use these functions in problems involving exponential growth and decay.

13.0 Students use the definition of logarithms to translate between logarithms in any base.

14.0 Students understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values.

15.0 Students determine whether a specific algebraic statement involving rational expressions, radical expressions, or logarithmic or exponential functions is sometimes true, always true, or never true.

16.0 Students demonstrate and explain how the geometry of the graph of a conic section (e.g., asymptotes, foci, eccentricity) depends on the coefficients of the quadratic equation representing it.

17.0 Given a quadratic equation of the form $ax^2 + by^2 + cx + dy + e = 0$, students can use the method for completing the square to put the equation into standard form and can recognize whether the graph of the equation is a circle, ellipse, parabola, or hyperbola. Students can then graph the equation.

18.0 Students use fundamental counting principles to compute combinations and permutations.

19.0 Students use combinations and permutations to compute probabilities.

20.0 Students know the binomial theorem and use it to expand binomial expressions that are raised to positive integer powers.

21.0 Students apply the method of mathematical induction to prove general statements about the positive integers.

22.0 Students find the general term and the sums of arithmetic series and of both finite and infinite geometric series.

23.0 Students derive the summation formulas for arithmetic series and for both finite and infinite geometric series.

24.0 Students solve problems involving functional concepts, such as composition, defining the inverse function and performing arithmetic operations on functions.

25.0 Students use properties from number systems to justify steps in combining and simplifying functions.

TRIGONOMETRY

1.0 Students understand the notion of angle and how to measure it, in both degrees and radians. They can convert between degrees and radians.

Note: **Bold** denotes an essential standard.

2.0 Students know the definition of sine and cosine as y- and x-coordinates of points on the unit circle and are familiar with the graphs of the sine and cosine functions.

3.0 Students know the identity $\cos^2(x) + \sin^2(x) = 1$:

3.1 Students prove that this identity is equivalent to the Pythagorean theorem (i.e., students can prove this identity by using the Pythagorean theorem and, conversely, they can prove the Pythagorean theorem as a consequence of this identity).

3.2 Students prove other trigonometric identities and simplify others by using the identity $\cos^2(x) + \sin^2(x) = 1$. For example, students use this identity to prove that $\sec^2(x) = \tan^2(x) + 1$.

4.0 Students graph functions of the form $f(t) = A \sin B(t + C)$ or $f(t) = A \cos B(t + C)$ and interpret A, B, and C in terms of amplitude, frequency, period, and phase shift.

5.0 Students know the definitions of the tangent and cotangent functions and can graph them.

6.0 Students know the definitions of the secant and cosecant functions and can graph them.

7.0 Students know that the tangent of the angle that a line makes with the x-axis is equal to the slope of the line.

8.0 Students know the definitions of the inverse trigonometric functions and can graph the functions.

9.0 Students compute, by hand, the values of the trigonometric functions and the inverse trigonometric functions at various standard points.

10.0 Students demonstrate an understanding of the addition formulas for sines and cosines and their proofs and can use those formulas to prove and/or simplify other trigonometric identities.

11.0 Students demonstrate an understanding of half-angle and double-angle formulas for sines and cosines and can use those formulas to prove and/or simplify other trigonometric identities.

12.0 Students use trigonometry to determine unknown sides or angles in right triangles.

13.0 Students know the law of sines and the law of cosines and apply those laws to solve problems.

14.0 Students determine the area of a triangle, given one angle and the two adjacent sides.

15.0 Students are familiar with polar coordinates. In particular, they can determine polar coordinates of a point given in rectangular coordinates and vice versa.

16.0 Students represent equations given in rectangular coordinates in terms of polar coordinates.

17.0 Students are familiar with complex numbers. They can represent a complex number in polar form and know how to multiply complex numbers in their polar form.

18.0 Students know DeMoivre's theorem and can give nth roots of a complex number given in polar form.

19.0 Students are adept at using trigonometry in a variety of applications and word problems.

MATHEMATICAL ANALYSIS

1.0 Students are familiar with, and can apply, polar coordinates and vectors in the plane. In particular, they can translate between polar and rectangular coordinates and can interpret polar coordinates and vectors graphically.

2.0 Students are adept at the arithmetic of complex numbers. They can use the trigonometric form of complex numbers and understand that a function of a complex variable can be viewed as a function of two real variables. They know the proof of DeMoivre's theorem.

3.0 Students can give proofs of various formulas by using the technique of mathematical induction.

4.0 Students know the statement of, and can apply, the fundamental theorem of algebra.

5.0 Students are familiar with conic sections, both analytically and geometrically.

Note: **Bold** denotes an essential standard.

5.1 Students can take a quadratic equation in two variables; put it in standard form by completing the square and using rotations and translations, if necessary; determine what type of conic section the equation represents; and determine its geometric components (foci, asymptotes, and so forth).

5.2 Students can take a geometric description of a conic section—for example, the locus of points whose sum of its distances from (1, 0) and (-1, 0) is 6—and derive a quadratic equation representing it.

6.0 Students find the roots and poles of a rational function and can graph the function and locate its asymptotes.

7.0 Students demonstrate an understanding of functions and equations defined parametrically and can graph them.

8.0 Students are familiar with the notion of the limit of a sequence and the limit of a function as the independent variable approaches a number or infinity. They determine whether certain sequences converge or diverge.

LINEAR ALGEBRA

1.0 Students solve linear equations in any number of variables by using Gauss-Jordan elimination.

2.0 Students interpret linear systems as coefficient matrices and the Gauss-Jordan method as row operations on the coefficient matrix.

3.0 Students reduce rectangular matrices to row echelon form.

4.0 Students perform addition on matrices and vectors.

5.0 Students perform matrix multiplication and multiply vectors by matrices and by scalars.

6.0 Students demonstrate an understanding that linear systems are inconsistent (have no solutions), have exactly one solution, or have infinitely many solutions.

7.0 Students demonstrate an understanding of the geometric interpretation of vectors and vector addition (by means of parallelograms) in the plane and in three-dimensional space.

8.0 Students interpret geometrically the solution sets of systems of equations. For example, the solution set of a single linear equation in two variables is interpreted as a line in the plane, and the solution set of a two-by-two system is interpreted as the intersection of a pair of lines in the plane.

9.0 Students demonstrate an understanding of the notion of the inverse to a square matrix and apply that concept to solve systems of linear equations.

10.0 Students compute the determinants of 2 x 2 and 3 x 3 matrices and are familiar with their geometric interpretations as the area and volume of the parallelepipeds spanned by the images under the matrices of the standard basis vectors in two-dimensional and three-dimensional spaces.

11.0 Students know that a square matrix is invertible if, and only if, its determinant is nonzero. They can compute the inverse to 2x2 and 3x3 matrices using row reduction methods or Cramer's rule.

12.0 Students compute the scalar (dot) product of two vectors in n-dimensional space and know that perpendicular vectors have zero dot product.

PROBABILITY AND STATISTICS

1.0 Students know the definition of the notion of independent events and can use the rules for addition, multiplication, and complementation to solve for probabilities of particular events in finite sample spaces.

2.0 Students know the definition of conditional probability and use it to solve for probabilities in finite sample spaces.

Note: **Bold** denotes an essential standard.

3.0 Students demonstrate an understanding of the notion of discrete random variables by using them to solve for the probabilities of outcomes, such as the probability of the occurrence of five heads in 14 coin tosses.

4.0 Students are familiar with the standard distributions (normal, binomial, and exponential) and can use them to solve for events in problems in which the distribution belongs to those families.

5.0 Students determine the mean and the standard deviation of a normally distributed random variable.

6.0 Students know the definitions of the mean, median, and mode of a distribution of data and can compute each in particular situations.

7.0 Students compute the variance and the standard deviation of a distribution of data.

8.0 Students organize and describe distributions of data by using a number of different methods, including frequency tables, histograms, standard line and bar graphs, stem-and-leaf displays, scatterplots, and box-and-whisker plots.

ADVANCED PLACEMENT PROBABILITY AND STATISTICS

1.0 Students solve probability problems with finite sample spaces by using the rules for addition, multiplication, and complementation for probability distributions and understand the simplifications that arise with independent events.

2.0 Students know the definition of conditional probability and use it to solve for probabilities in finite sample spaces.

3.0 Students demonstrate an understanding of the notion of discrete random variables by using this concept to solve for the probabilities of outcomes, such as the probability of the occurrence of five or fewer heads in 14 coin tosses.

4.0 Students understand the notion of a continuous random variable and can interpret the probability of an outcome as the area of a region under the graph of the probability density function associated with the random variable.

5.0 Students know the definition of the mean of a discrete random variable and can determine the mean for a particular discrete random variable.

6.0 Students know the definition of the variance of a discrete random variable and can determine the variance for a particular discrete random variable.

7.0 Students demonstrate an understanding of the standard distributions (normal, binomial, and exponential) and can use the distributions to solve for events in problems in which the distribution belongs to those families.

8.0 Students determine the mean and the standard deviation of a normally distributed random variable.

9.0 Students know the central limit theorem and can use it to obtain approximations for probabilities in problems of finite sample spaces in which the probabilities are distributed binomially.

10.0 Students know the definitions of the mean, median, and mode of distribution of data and can compute each of them in particular situations.

11.0 Students compute the variance and the standard deviation of a distribution of data.

12.0 Students find the line of best fit to a given distribution of data by using least squares regression.

Note: **Bold** denotes an essential standard.

13.0 Students know what the correlation coefficient of two variables means and are familiar with the coefficient's properties.

14.0 Students organize and describe distributions of data by using a number of different methods, including frequency tables, histograms, standard line graphs and bar graphs, stem-and-leaf displays, scatterplots, and box-and-whisker plots.

15.0 Students are familiar with the notions of a statistic of a distribution of values, of the sampling distribution of a statistic, and of the variability of a statistic.

16.0 Students know basic facts concerning the relation between the mean and the standard deviation of a sampling distribution and the mean and the standard deviation of the population distribution.

17.0 Students determine confidence intervals for a simple random sample from a normal distribution of data and determine the sample size required for a desired margin of error.

18.0 Students determine the P-value for a statistic for a simple random sample from a normal distribution.

19.0 Students are familiar with the chi-square distribution and chi-square test and understand their uses.

CALCULUS

1.0 Students demonstrate knowledge of both the formal definition and the graphical interpretation of limit of values of functions. This knowledge includes one-sided limits, infinite limits, and limits at infinity. Students know the definition of convergence and divergence of a function as the domain variable approaches either a number or infinity:

1.1 Students prove and use theorems evaluating the limits of sums, products, quotients, and composition of functions.

1.2 Students use graphical calculators to verify and estimate limits.

1.3 Students prove and use special limits, such as the limits of $(sin(x))/x$ and $(1-cos(x))/x$ as x tends to 0.

2.0 Students demonstrate knowledge of both the formal definition and the graphical interpretation of continuity of a function.

3.0 Students demonstrate an understanding and the application of the intermediate value theorem and the extreme value theorem.

4.0 Students demonstrate an understanding of the formal definition of the derivative of a function at a point and the notion of differentiability:

4.1 Students demonstrate an understanding of the derivative of a function as the slope of the tangent line to the graph of the function.

4.2 Students demonstrate an understanding of the interpretation of the derivative as an instantaneous rate of change. Students can use derivatives to solve a variety of problems from physics, chemistry, economics, and so forth that involve the rate of change of a function.

4.3 Students understand the relation between differentiability and continuity.

4.4 Students derive derivative formulas and use them to find the derivatives of algebraic, trigonometric, inverse trigonometric, exponential, and logarithmic functions.

5.0 Students know the chain rule and its proof and applications to the calculation of the derivative of a variety of composite functions.

Note: **Bold** denotes an essential standard.

6.0 Students find the derivatives of parametrically defined functions and use implicit differentiation in a wide variety of problems in physics, chemistry, economics, and so forth.

7.0 Students compute derivatives of higher orders.

8.0 Students know and can apply Rolle's theorem, the mean value theorem, and L'Hôpital's rule

9.0 Students use differentiation to sketch, by hand, graphs of functions. They can identify maxima, minima, inflection points, and intervals in which the function is increasing and decreasing.

10.0 Students know Newton's method for approximating the zeros of a function.

11.0 Students use differentiation to solve optimization (maximum-minimum problems) in a variety of pure and applied contexts.

12.0 Students use differentiation to solve related rate problems in a variety of pure and applied contexts.

13.0 Students know the definition of the definite integral by using Riemann sums. They use this definition to approximate integrals.

14.0 Students apply the definition of the integral to model problems in physics, economics, and so forth, obtaining results in terms of integrals.

15.0 Students demonstrate knowledge and proof of the fundamental theorem of calculus and use it to interpret integrals as antiderivatives.

16.0 Students use definite integrals in problems involving area, velocity, acceleration, volume of a solid, area of a surface of revolution, length of a curve, and work.

17.0 Students compute, by hand, the integrals of a wide variety of functions by using techniques of integration, such as substitution, integration by parts, and trigonometric substitution. They can also combine these techniques when appropriate.

18.0 Students know the definitions and properties of inverse trigonometric functions and the expression of these functions as indefinite integrals.

19.0 Students compute, by hand, the integrals of rational functions by combining the techniques in standard 17.0 with the algebraic techniques of partial fractions and completing the square.

20.0 Students compute the integrals of trigonometric functions by using the techniques noted above.

21.0 Students understand the algorithms involved in Simpson's rule and Newton's method. They use calculators or computers or both to approximate integrals numerically.

22.0 Students understand improper integrals as limits of definite integrals.

23.0 Students demonstrate an understanding of the definitions of convergence and divergence of sequences and series of real numbers. By using such tests as the comparison test, ratio test, and alternate series test, they can determine whether a series converges.

24.0 Students understand and can compute the radius (interval) of the convergence of power series.

25.0 Students differentiate and integrate the terms of a power series in order to form new series from known ones.

26.0 Students calculate Taylor polynomials and Taylor series of basic functions, including the remainder term.

27.0 Students know the techniques of solution of selected elementary differential equations and their applications to a wide variety of situations, including growth-and-decay problems.

Note: **Bold** denotes an essential standard.